Dual Dig Level I (2012)

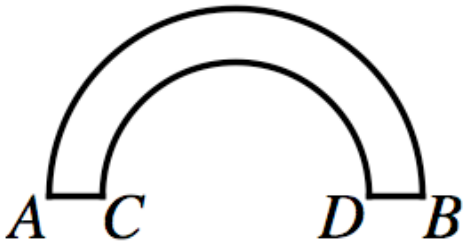
1. The 100 students at a school must take a math class or a science class this term; some take both. This term, if 55 of the students are taking a math class, and if 20 of the students are taking both math and science classes, how many of the students are taking a science class?

2. What is the average of $\frac{1}{6}$ and $\frac{1}{8}$?

3. Given that $x < 0$, simplify: $\sqrt{x^2} + x$

4. Rick gives his favorite integer to Larry and Ken. Larry squares Rick's integer, Ken doubles Rick's integer, and they both give their results to Tammi. When Tammi adds the results from Larry and Ken, she gets the cube of Rick's integer. What are the three possibilities for Rick's integer?

5. The figure below represents an audience pit. The boundary of the pit consists of two semicircles and two line segments. The two semicircles are the upper halves of two concentric circles (i.e., circles with the same center). The semicircle $\overset{\frown}{AB}$ is the upper half of a circle of radius 7 yards. The semicircle $\overset{\frown}{CD}$ is the upper half of a circle of radius 5 yards. Find the total perimeter of the boundary of the pit.



6. A square is formed by joining the midpoints of the sides of a larger square. The area of the smaller square will then be what fraction of the area of the larger square?

7. What is the smallest positive integer that is divisible by 2, 3, 4, 5, 6, 7, 8, 9, and 10?

8. Solve for *x*: $\frac{1}{81}\left(9^x\right)^{x+4} = 81^{2x+7}$

9. Consider Triangle *MAS* with $\overline{MA} \perp \overline{AS}$, with point *E* somewhere on $\overline{MS}$. A new line segment is drawn from *A* to *E* such that $\overline{AE} \perp \overline{MS}$. Given that *ME* = 5 inches, and *AE* = 7 inches, find the length of $\overline{ES}$.

10. Find all real solutions of: $\left| x^3 - 8 \right| > 0$

11. Find the equation of the parabola in the usual *xy*-plane that contains the points (1, 8), (0, 7), and $(-2, 23)$.

12. Simplify completely: $\left(\frac{-36x^{-3}y^{-7}z^{6}}{24x^{-4}y^{-5}z^{-2}}\right)^{-2}\left(\frac{24x^{-2}y^{3}z^{6}}{4z^{12}}\right)^{0}\left(\frac{3x^{5}y^{4}}{z^{-2}}\right)^{3}$. Assume *x*, *y*, and *z* are nonzero.

13. Determine the points in the usual *xy*-plane where the circle $x^2 + y^2 = 16$ and the parabola $y = x^2 - 4$ intersect.

14. Simplify completely: $\left(\frac{\log_5 125}{\log_5 9}\right)\left(\frac{\log_5 81}{\log_2 \sqrt[5]{5}}\right)\left(\frac{\log_2 125}{\log_b b^3}\right)$

15. Find all real solutions of the equation: $2\left(x^2-3\right)^2 + 3\left(x^2-3\right) = 4x^2 - 12$

16. Five pennies are flipped independently of one another. Assume that each penny is fair; a 'head' or a 'tail' is equally likely. What is the probability that at least one of the pennies will come up 'heads'?

17. If $f(x) = 5 + x - x^2$, find $\frac{f(a+h) - f(a)}{h}$ in completely simplified form. Assume $h \neq 0$.

18. Find all real solutions of the inequality: $\frac{x^2}{2x-3} > 3$

19. Solve for *y* in terms of *x*: $9x^2 - 4y^2 = 25 - 16y$

20. What digit is in the ones place of 7^{2012}?